Study of Whispering Gallery Modes in Double Disk Sapphire Resonators

Hong Peng

Abstract—**A mode matching method for determining the resonant frequencies of the whispering gallery modes in a double cylindrical disk dielectric resonator is presented. The method has an improved accuracy in the calculation of the transverse magnetic modes with high axial numbers in a single uniaxial-anisotropic disk resonator. In this paper, frequency tuning and detuning due to the interaction between various whispering gallery modes will also be discussed.**

I. Introduction

HIGH PURITY sapphire dielectric resonators have been used to construct low noise and ultra stable oscillators owing to their very low dielectric loss and low thermal expansion coefficient [1]–[3]. A double disk sapphire resonator with a high Q-factor and frequency tuning coefficient has also been employed to construct an ultra-high sensitivity vibration transducer [4], [5].

Accurate determination of the resonant frequencies of whispering gallery (WG) modes from complete field solutions for the tunable dielectric resonators which comprise two or more pieces of dielectrics is very difficult, and there is a need to find simplified approaches. A variety of the tunable dielectric resonators with two or more pieces of dielectrics and methods of calculating their resonant frequencies (low order modes) have been reported previously [6]–[9]. One such method is the mode matching method which solves the basic electromagnetic field equations with relevant boundary conditions [7], [9]. This method can usually provide accurate solutions for some simple and symmetrical resonator configurations. There is also the coupling method which uses equivalent electric circuits to model the coupled resonators [8], [10]. It provides a simple way to calculate the resonant frequency tuning and detuning due to the interaction of two coupled modes. But this coupling calculation method does not directly give the resonant frequencies, and is limited to coupled modes with simple coupling coefficients. (e.g., linear or constant coupling coefficients) When the coupling coefficients are neither linear nor constant, the coupling calculation becomes quite difficult. A finite element method has been introduced to calculate the resonant frequencies of a temperature compensated sapphire resonator [11]. It is expected to be capable of solving problems of the resonators with complex configurations.

In this paper, an improved mode matching method for determining the resonant frequencies of WG modes in double cylindrical dielectric disk resonators is presented in Section II. This is an extension of Garault and Guillon's method [12], [13] from one piece of isotropic dielectric to two pieces of anisotropic dielectric. The method is applied to quasi-TE and quasi-TM WG modes with even or odd axial mode numbers. By taking account of the different axial propagation and decay constants for TM and TE modes inside and outside the dielectric due to uniaxial anisotropy, the method allows the calculation for the TM modes with high axial number to be more accurate than previous work [14]. The theoretical calculation of the resonant frequencies is consistent with experimental results obtained from the double disk resonators [15], even for high axial mode numbers.

In Section III, a study of the interaction between WG modes in the double dielectric disk resonator is presented. A strong coupling between WG modes can cause their resonant frequency tuning and detuning and degrades the accuracy of the mode matching method. The mode transition and the coupling mechanism of the WG modes in the double dielectric disk resonators is also discussed.

II. Resonant Frequencies of the Whispering Gallery Modes

A. Theory

Whispering gallery (WG) modes are the hybrid type modes. However as a first approximation we can consider that there are two types of WG modes designated quasi-$\mathrm{TM}_{mnp}(\mathrm{WGH}_{mnp+1})$ modes and quasi-$\mathrm{TE}_{mnp}(\mathrm{WGE}_{mnp+1})$ modes. Each mode is denoted by three mode numbers m, n and p describing the number of field variations along the azimuthal, radial, and axial directions respectively.

A cylindrical double disk resonator model with defined cylindrical coordinates is illustrated by Fig. 1. To simplify the model, the two dielectric disks are assumed to be identical and are standing in free space. The dielectric is uniaxial anisotropic and its crystallographic c-axis is assumed to be parallel to the z direction. The permittivity of the dielectric parallel the z-axis is defined as ε_z. The permittivity perpendicular to the z-axis defined as ε_r, with $\varepsilon_r = \varepsilon_\phi$. The resonator is divided into 8 regions which are labeled by 1, 2, 3, 4, 1', 2' 3' and 4'. Owing to its symmetry configuration, only four regions (1,

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The author was with the University of Western Australia, Department of Physics, Nedlands, WA 6009, Australia. He is now with the Physics Department, Massachusetts Institute of Technology, Cambridge, MA 02139 USA.

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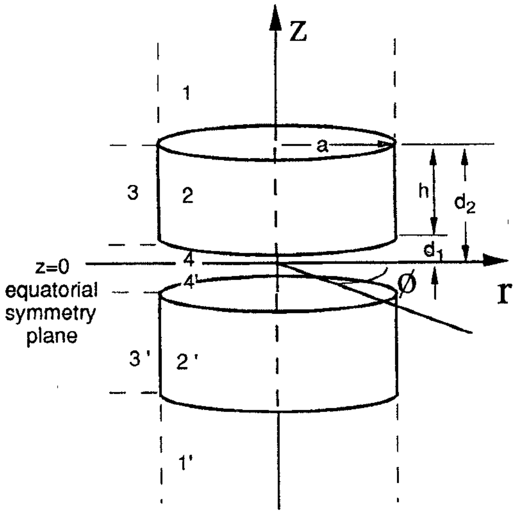

Fig. 1. Cylindrical coordinates for the double disk resonator in free space. The two disks are identical.

2, 3, and 4) above the symmetry plane are considered in the analysis.

The determination of resonant frequencies of the WG modes requires solution of Maxwell's equations with appropriate boundary conditions. The propagation characteristics of the electrical and the magnetic fields depend on the configuration of the double disk resonator. Inside the dielectric the fields are spatially periodic, but the fields are evanescent in regions 1, 3 and 4 outside the dielectric. Assuming the arguments for the z dependence of components E_z and H_z in regions 2 and 3 are equal, the z components E_z and H_z for the resonator can be written as

$$E_{z1} = A_E J_m(k_E r)\cos(m\phi)\exp(-\alpha_E z), \tag{1a}$$
$$E_{z2} = B_E J_m(k_E r)\cos(m\phi)[A_1 \sin(\beta z) + B_1 \cos(\beta z)], \tag{1b}$$
$$E_{z3} = C_E K_m(k_3 r)\cos(m\phi)[A_1 \sin(\beta z) + B_1 \cos(\beta z)], \tag{1c}$$
$$E_{z4} = D_E J_m(k_E r)\cos(m\phi)\begin{Bmatrix}\sinh(\alpha_E z), \\ \cosh(\alpha_E z)\end{Bmatrix}, \tag{1d}$$
$$H_{z1} = A_H J_m(k_H r)\sin(m\phi)\exp(-\alpha_H z), \tag{1e}$$
$$H_{z2} = B_H J_m(k_H r)\sin(m\phi)[A_2 \sin(\beta z) + B_2 \cos(\beta z)], \tag{1f}$$
$$H_{z3} = C_H K_m(k_3 r)\sin(m\phi)[A_2 \sin(\beta z) + B_2 \cos(\beta z)], \tag{1g}$$
$$H_{z4} = D_H J_m(k_H r)\sin(m\phi)\begin{Bmatrix}\cosh(\alpha_H z), \\ \sinh(\alpha_H z)\end{Bmatrix} \tag{1h}$$

where $k_E^2 = \varepsilon_z k_0^2 - \beta^2 \varepsilon_z/\varepsilon_r, k_H^2 = \varepsilon_r k_0^2 - \beta^2, k_3^2 = \beta^2 - k_0^2, k_E^2 = k_0^2 + \alpha_E^2$, and $k_H^2 = k_0^2 + \alpha_H^2$. Here m is the azimuthal mode number, β the axial propagation constant, k_0 the free space wave number which has $k_0^2 = \omega^2 \mu_0 \varepsilon_0, k_3$ the radial propagation constant outside the dielectric, k_E the radial propagation constant inside the dielectric for the electrical field, k_H the radial propagation constant inside the dielectric for the magnetic field, α_E and α_H the axial decay constants outside the dielectric for the electrical field and the magnetic field respectively. The transverse components can be obtained from the components E_z and H_z based on the Maxwell equations [13].

The radial matching condition that requires equality of the tangential components of the E and H fields at $r = \mathrm{a}$ on the boundary of region 2 and region 3 yields a transcendental equation as

$$\left(\frac{K_m'(\xi)}{\xi K_m(\xi)} + \frac{\varepsilon_r \lambda_E J_m'(\lambda_E)}{\lambda_H^2 J_m(\lambda_E)}\right)\left(\frac{K_m'(\xi)}{\xi K_m(\xi)} + \frac{J_m'(\lambda_H)}{\lambda_H J_m(\lambda_H)}\right) = \frac{m^2(\xi^2 + \lambda_H^2)(\varepsilon_r \xi^2 + \lambda_H^2)}{\xi^4 \lambda_H^4} \tag{2}$$

where $\xi = k_3\mathrm{a}, \lambda_H = k_H\mathrm{a}$ and $\lambda_E = k_E\mathrm{a}$. When $k_3^2 < 0, \xi$ is purely imaginary and the Bessel function K_m in equation (2) is replaced with a Hankel function of the second kind $H_m^{(2)}$.

For the axial matching, the transverse electric (TE) modes and the transverse magnetic (TM) modes in regions 1, 2, and 4 must be considered. The components E_z and H_z are assumed zero for quasi-TE and quasi-TM modes respectively. By satisfying the axial boundary conditions that the transverse electric and magnetic fields be continuous at the plane interfaces $z = d_1$ and $z = d_2$, four transcendental equations are obtained

TE_{mnp}, p even;

$$(-\alpha_H/\beta + \tan(\beta d_2))(1 - \alpha_H/\beta \tanh(\alpha_H d_1)\tan(\beta d_1)) = (1 + \alpha_H/\beta \tan(\beta d_2))(\alpha_H/\beta \tanh(\alpha_H d_1) + \tan(\beta d_1)), \tag{3a}$$

p odd

$$(-\alpha_H/\beta + \tan(\beta d_2))(1 - \alpha_H/\beta \coth(\alpha_H d_1)\tan(\beta d_1)) = (1 + \alpha_H/\beta \tan(\beta d_2))(\alpha_H/\beta \coth(\alpha_H d_1) + \tan(\beta d_1)), \tag{3b}$$

TM_{mnp}, p even

$$(-\varepsilon_r\alpha_E/\beta + \tan(\beta d_2))(1 - \varepsilon_r\alpha_E/\beta \tanh(\alpha_E d_1)\tan(\beta d_1)) = (1 + \varepsilon_r\alpha_E/\beta \tan(\beta d_2))(\varepsilon_r\alpha_E/\beta \tanh(\alpha_E d_1) + \tan(\beta d_1)), \tag{3c}$$

p odd

$$(-\varepsilon_r\alpha_E/\beta + \tan(\beta d_2))(1 - \varepsilon_r\alpha_E/\beta \coth(\alpha_E d_1)\tan(\beta d_1)) = (1 + \varepsilon_r\alpha_E/\beta \tan(\beta d_2))(\varepsilon_r\alpha_E/\beta \coth(\alpha_E d_1) + \tan(\beta d_1)). \tag{3d}$$

The above transcendental equations (2) and (3) can be solved numerically using a computer. The resonant frequencies of WG modes for a variable gap spacing $2d_1$ can then be obtained. The above analysis for the double dielectric disk resonator can be extend to an ordinary one-piece cylindrical resonator when the gap spacing x between two disks becomes zero (i.e., $x = 2d_1 = 0$).

B. Experimental and Theoretical Results

A double disk sapphire resonators was tested in free space at room temperature to verify the theory. The cylindrical sapphire resonator has a radius $\mathrm{a}_1 = \mathrm{a}_2 = 15.81$ mm and height $h_1 = 14.42$ mm and $h_2 = 14.44$ for the two disks respectively.

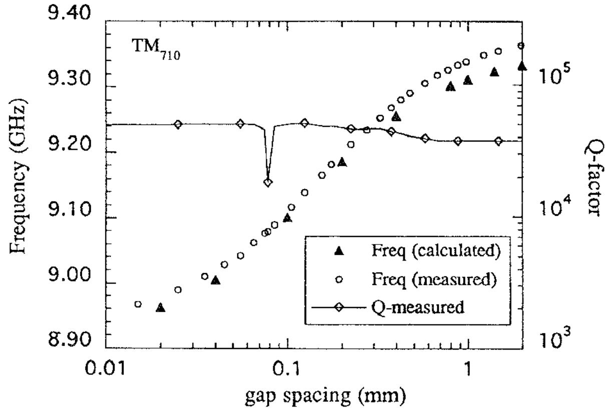

Fig 2. The measured and calculated resonant frequencies and Q-factor of the TM_{710} mode in the resonator as a function of the gap spacing.

TABLE I
RESONANT FREQUENCIES OF SOME WG MODES IN THE DOUBLE DISK SAPPHIRE RESONATOR (a = 15 8 mm, h = 14 4 mm)

gap spacing	0		0.1 mm		1 mm	
modes	theory	experi	theory	experi	theory	expri
TM_{710}	8.910	8.933	9.101	9.110	9.312	9.341
TM_{711}	9.319	9.370	9.319	9.372	9.318	9.377
TM_{712}	10.076	10.142	10.314		10.838	10.799
TM_{713}	10.969	11.059	10.969	11.071	10.961	11.156
TE_{610}	9 6836	9.702	9.7145	9.722	9.901	9.822
TE_{611}	10.141	10.126	10.141	10.156	10.116	10.296
TE_{710}	10.856	10.828	10.895	10.848	11 050	10 977
TE_{711}	11.235	11.207	11.235	11.230	11.210	11.290

The average height of the disks, $h = (h_1+h_2)/2$, was used for the calculation. The sapphire dielectric constants used for the calculation are the same as given by Shelby [16], $\varepsilon_z = 11.589$ and $\varepsilon_r = \varepsilon_\phi = 9.395$.

The WG modes of the double disk resonator were experimentally investigated when the gap spacing between two disks varied. Fig. 2 shows the experimental and theoretical results of the TM_{710} mode in the resonator as the gap spacing varies. The circular points in Fig. 2 represents a typical resonant frequency tuning curve for the high tuning coefficient WG modes. The measurements are in good agreement with theoretical values. As the gap spacing increases above 0.3 mm, the difference between theoretical and experimental values increases. But the difference is still less than 0.4% at $x = 2$ mm. Fig. 2 also shows that the Q-factor of mode TM_{710} in the resonator is nearly independent of the gap spacing variations. The degradation of the Q-factor at the gap spacing of $\sim$0.1 mm is due to the interaction of the TM_{710} mode with the TE_{511} mode.

Table I shows more theoretical and experimental results of WG modes in the resonator at three different gap spacing. Experimental results and theoretical calculation show that the gap spacing variation has a strong influence on some TM_{mnp} modes whose axial number p is zero or an even integer. Fig. 3 shows the calculated and measured results of the WG modes with three different azimuthal numbers as the gap spacing varies. As the gap spacing increases, the TM modes with zero or even p have a transition toward TM modes with

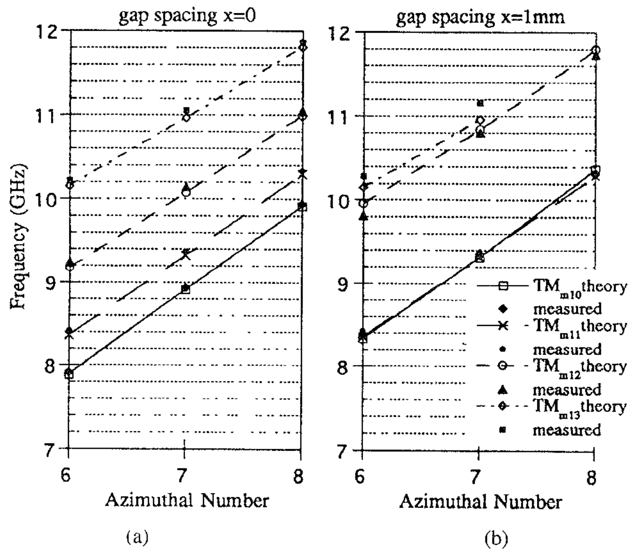

Fig. 3. Comparison of the theoretical and experimental resonant frequencies of the TM modes as the gap spacing has two different values. (a) $x = 0$; (b) $x = 1$ mm.

axial numbers $p + 1$ which are also compatible with the boundary conditions for one-piece disk resonator. This is why the TM modes with odd axial numbers have less dependence on the gap spacing. Compared with the large frequency tuning TM modes, the resonant frequencies of TE modes show less dependence on the variation of the gap spacing. This is because the large frequency tuning TM modes have a higher field density in the gap which is mainly due to the field discontinuity of their E_z components on the plane boundary of the dielectric. The components of TE modes however remain unchanged on the boundary. Like the TM modes, the discrepancy between experimental and theoretical results for TE modes increases with an increase of the gap spacing. This is due to the fact that both the TM and TE modes are hybrid modes and a more sophisticated theory is required for double resonators with a large gap spacing.

When the double resonator becomes a single resonator ($x = 0$), experiment and calculation agree to a high accuracy, a few tenths of a percent for the fundamental modes as shown in Table I. In comparison with Tobar's paper [14], the theory gives improved accuracy ($<1\%$) for calculation of the TM modes with high axial mode number ($p \geq 2$) in single resonators. The TM and TE modes have different axial propagation and decay constants inside and outside the dielectric due to uniaxial anisotropy, which explains the discrepancy between theoretical and experimental results for TM modes with high axial numbers in Tobar's paper. The errors between theory and measurements for the single resonators is probably due to uncertainties in permittivity and dimensions.

III. INTERACTION BETWEEN WHISPERING GALLERY MODES

In most cases, the calculations using the mode matching method have shown good agreement with experiment. However there is an exception when a WG mode interacts strongly with another WG mode, because the resonant frequencies of the two coupled modes are detuned by each other.

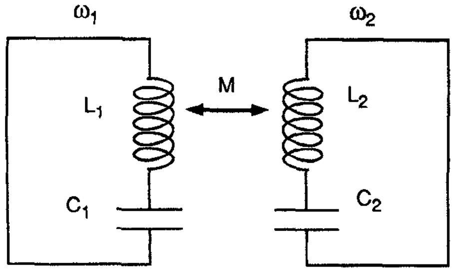

Fig. 4. An equivalent circuit model of two inductively coupled modes.

The interaction between two coupled WG modes can cause the variation of the mode frequencies, their Q-factors or both. From an equivalent circuit model point of view, there are two types of interaction: resistive coupling and inductive (or reactance) coupling. When an interaction changes the coupled mode frequencies largely, this interaction may be called inductive coupling (or reactive coupling). If an interaction mainly changes the Q-factors of the coupled modes, this interaction may be called resistive coupling. Based on observed interaction effects in the isolated double disk resonator, most if not all interaction between two coupled WG modes fall into these two categories.

For the resistive coupling, the coupling between two WG modes is weak. The resonant frequencies of the coupled modes remain unaffected or the frequency detuning is negligible. For the inductive coupling, the coupling between two WG modes is usually strong. The coupled modes do not *tune across* each other. The stronger the coupling, the larger the separation of the two coupled normal modes. In this case, there is a fully mutual mode transition between the two coupled modes if the two modes are different WG modes.

A. Modeling of Inductive Coupling

A simple equivalent circuit model for two inductively coupled modes in a double disk dielectric resonator is illustrated in Fig. 4. It consists of two series resonant circuits coupled by a mutual inductance. Here the losses of the resonators and the coupling due to probes or cavities are ignored. A coupling coefficient κ is defined in terms of the resonant circuit parameters by

$$\kappa = \frac{M}{\sqrt{L_1 L_2}} \tag{4}$$

where M is the mutual inductance of the two resonant circuits, L_1 and L_2 are the inductance of the two resonant circuits, respectively. The two resonators have uncoupled resonant frequencies $\omega_1 = 1/\sqrt{L_1 C_1}$ and $\omega_2 = 1/\sqrt{L_2 C_2}$ respectively, and assuming $\omega_1 > \omega_2$. The frequencies of normal modes in the coupled resonators is given by

$$\omega_\pm^2 = \frac{\omega_1^2 + \omega_2^2 \pm \sqrt{(\omega_1^2 - \omega_2^2)^2 + 4\kappa^2\omega_1^2\omega_2^2}}{2(1-\kappa^2)} \tag{5}$$

where ω_+ and ω_- are defined as the coupled normal modes. Assuming $2(\omega_1^2 - \omega_2^2)/(\omega_1^2 + \omega_2^2) \ll 1$, (5) can be simplified and written as

$$\omega_\pm^2 \approx \omega_0^2(1 \pm \kappa), \quad \text{when} \quad \kappa > \frac{\Delta\omega^2}{2\omega_0^2} \tag{6}$$

$$\omega_\pm^2 \approx \omega_1^2 \pm \frac{\omega_0^4}{\Delta\omega^2}\kappa^2, \quad \text{when} \quad \kappa < \frac{\Delta\omega^2}{2\omega_0^2} \tag{7}$$

where $\omega_0^2 = (\omega_1^2 + \omega_2^2)/2$ and $\Delta\omega^2 = \omega_1^2 - \omega_2^2$. Under the condition of $\kappa > \Delta\omega^2/(2\omega_0^2)$, the coupling coefficient κ can be simply derived from (4) using $\omega\pm$ which can be measured

$$\kappa = \frac{\omega_+^2 - \omega_-^2}{\omega_+^2 + \omega_-^2}. \tag{8}$$

As shown in (6) and (7), the inductive coupling will cause a separation between two normal modes, and they cannot be tuned to cross each other.

B. Comparison with Experimental Results

Two typical inductive couplings in the double disk resonators are observed: 1) coupling between two identical modes whose uncoupled resonant frequencies have the same or nearly the same values 2) coupling between two different modes whose uncoupled resonant frequencies cannot be tuned to cross each other.

Fig. 5 shows a typical coupling between two identical modes in the double disk resonator. When the gap spacing decreases from a large value, the resonant frequency for one disk increases and the resonant frequency for the other disk decreases. Assuming the coefficient $1/\kappa$ is proportional to the gap spacing, the theoretical and experimental results are in good agreement when the gap spacing has a large value. But this assumption is not true when the gap spacing has a small value and two modes become decoupled. In this case, the mode frequency tuning is essentially attributed to the change of the boundary conditions in the double disk resonator which can be predicted using the theory described in Section II. The mode transition accounts for the difference between the value f-predicted using the inductive model and the measured results in Fig. 6 when the gap spacing is less than 4 mm. In practice, the frequency tuning due to inductive coupling between the same WG modes can be observed only when the frequency difference of the two uncoupled modes is very small. Otherwise the interaction between the two modes does not occur.

A typical coupling between two different modes is shown in Fig. 6. The coupling has a tuning characteristic where the coupled modes do not tune across each other but they exchange their identities. In this case, the experimental and theoretical results are in good agreement. In the calculation, the coupling coefficient is assumed to be a constant which is true for the interaction in a very small range of the gap spacing. The coupling coefficient in this case is experimentally determined from equation (8), $\kappa \approx 0.006$. The observed gap spacing dependence of the Q-factors of the two coupled modes is more complicated and beyond the scope of the equivalent circuit model.

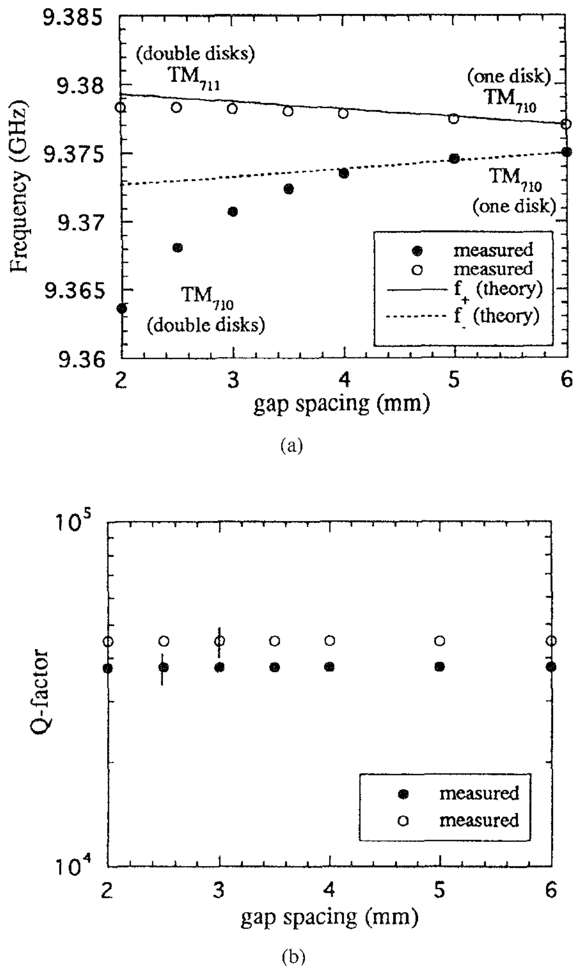

Fig. 5. (a) Experimental results show a typical resonant frequency tuning of two identical TM modes The theoretical and experimental results have a good agreement when the gap spacing has a large value (b) Measured Q-factors of the two TM modes

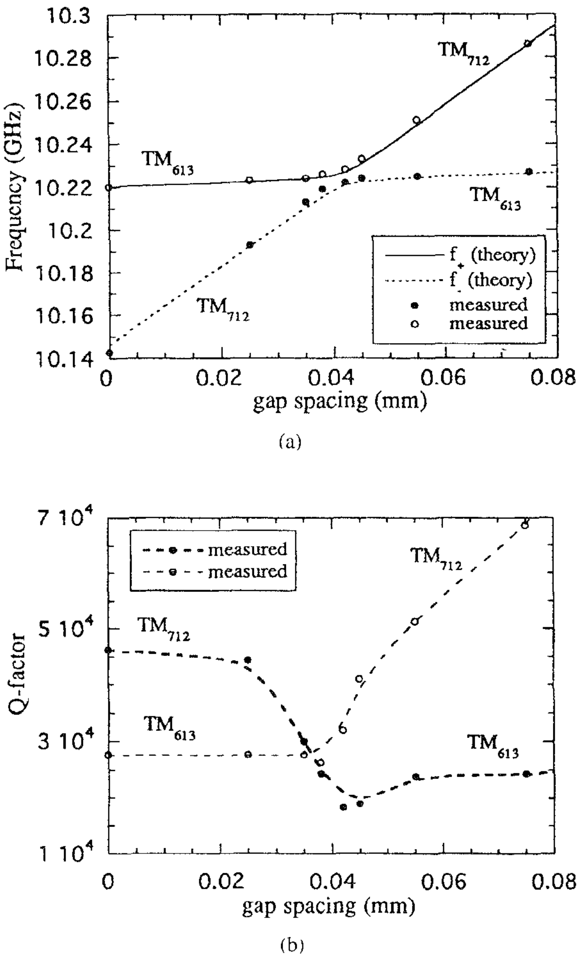

Fig. 6. (a) A typical inductive coupling between two different TM modes (b) Measured Q-factors of the two TM modes.

C. Coupling Rules and Mechanism

The WG modes are not simply orthogonal eigen-frequency modes. They are hybrid modes and the interaction between the coupled WG modes in the double disk resonators has some interesting characteristics. The coupling between coupled WG modes with the same radial number $n = 1$ in double disk resonators were experimentally investigated. For isolated double disk resonators, the experiments showed that there are some selection rules for the inductively coupled modes. The inductive coupling between two WG modes occurs only when the differences of the azimuthal and axial mode numbers satisfy the following relations,

$$\Delta p = 0, \pm 1 \tag{9}$$

$$\Delta m = 0, \pm 1 \tag{10}$$

where Δp and Δm are axial and azimuthal number differences of the two WG modes respectively. In isolated double disk resonators, the inductive coupling occurs only between quasi-TM modes or between quasi-TE modes. There is no inductive coupling between TM and TE modes. As shown in Fig. 6, the tuning curves with an inductive coupling show a mode transition when the gap spacing increases, i.e., mode TM_{712} changes into TM_{613} and mode TM_{613} becomes TM_{712} after the interaction. When the two modes are strongly coupled, they become hybrids of each other. The reactive coupling of two modes can occur in a wide frequency range which is larger than their bandwidth.

A typical resistive coupling is illustrated in Fig. 7. The coupling only influences the Q-factor of the high Q TM_{710} mode. This resistive coupling can also occur between TM modes in isolated double disk resonators. Usually resistive coupling occurs inside the bandwidth of the modes. The resonant frequencies of the coupled modes remain unaffected.

The above coupling rules are only true for isolated double disk resonators. When the double disk resonators are shielded inside metal cavities, the coupling is modified and become stronger in comparison with the isolated case. In this case, inductive coupling can occur between more sets of modes. For example, the interaction between the TM_{710} and TE_{511} *in a cavity* appears to be a inductive coupling and their coupling coefficient $\kappa \approx 0.0007$. However, the modes having close mode numbers, especially the mode numbers which satisfy the relations (9) and (10), show stronger couplings than the others.

The observed phenomenon of the interaction suggests that the couplings between the coupled WG modes are mainly at-

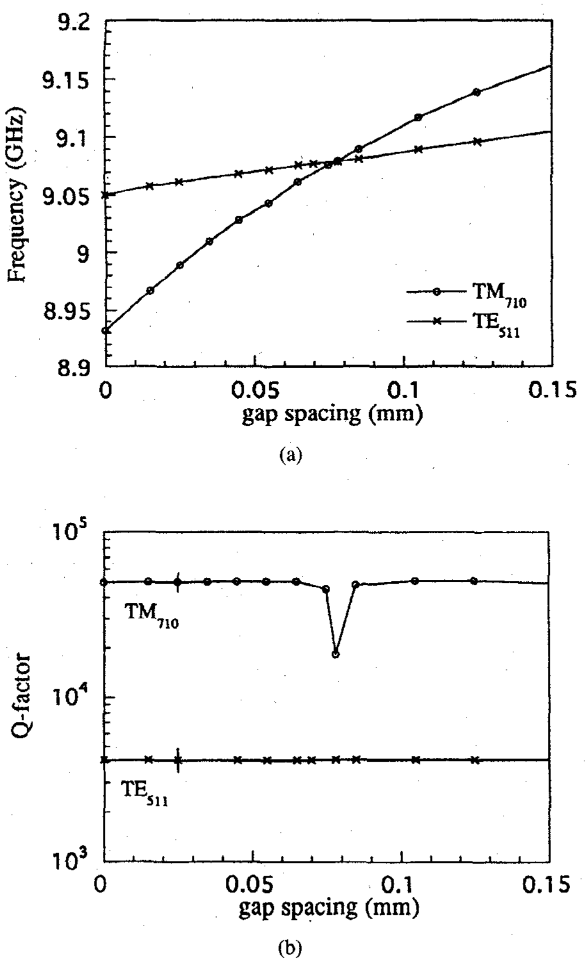

Fig. 7. The interaction between two resistively coupled modes. (a) Measured frequencies. (b) Measured Q-factors of the two TM modes.

tributed to their electromagnetic field states. This is because the modes with close mode numbers have similar electromagnetic field states that provide large field overlap (or interaction cross section) and thus strong couplings. (This phenomenon may be considered to be analogous to the state transition of atoms in which the transition occurs only between closer states.)

IV. Conclusion

A mode matching method for determining the resonant frequencies of the WG modes in double disk resonators has been shown to give frequencies accurate to a few tenths of a percent for the fundamental WG modes. The theory gives improved accuracy ($<1\%$) for calculation of the TM modes with high axial mode number ($p \geq 2$) in single resonators. Experiments and theory have both shown that the TM modes with a zero or even axial mode number in the double disk resonators have larger frequency tuning than other modes .

The interaction between various WG modes in the double disk resonator has been extensively investigated. The coupling of various WG modes is mainly determined by their electromagnetic field states. The closer the mode states, the stronger the coupling of the coupled WG modes. Mode transition may occur between the strongly coupled WG modes.

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Hong Peng was born in Kunming, China, in 1963. He received the B.S. and M.S. degrees, both in physics, in 1984 and 1987, respectively, from Zhongshan University, Guangzhou, China. He received the Ph.D. degree in physics from the University of Western Australia in 1996. His doctoral dissertation involved the development of ultra-high sensitive sapphire resonator transducers.

Since December 1995, he has been with Massachusetts Institute of Technology as a Postdoctoral Associate. Currently his interests are in the detection of dark matter axion, low noise amplifiers, and cryogenic resonators.